Probe-Shift Error in Remote Diagnostic of Volume Radiation Sources

C. J. Cremers* and P. S. Swathi†
University of Kentucky, Lexington, Kentucky

Determination of the distribution of the emission coefficient in a volume radiation source sometimes entails mathematical analysis of an externally measured distribution of radiative flux. Through an analysis of a hypothetical source exhibiting cylindrical symmetry, it is shown that the finite aperture of the measuring device causes a probe-shift error in the results. The shift is a strong function of the size and magnitude of the second spatial derivative of the emission and increase with the aperture size.

Nomenclature

$(A(y_l, w, h)$ = measured radiative power, W
$B(\bar{y})$ = calculated radiative flux, W/m^2
h = aperture height of analyzer, m
$i(r)$ = emission coefficient, W/m^3
$I(y)$ = ideal radiative flux, W/m^2
r = radial coordinate, m
R = radius of source, m
w = aperture width of analyzer, m
x = depth coordinate normal to source axis, m
y = lateral coordinate normal to source axis, m
$\bar{y}$ = effective value of lateral coordinate, m
y_l = nominal value of lateral coordinate, m
z = axial coordinate of source , m
σ = standard deviation of Gauss distribution, m

Introduction

THERE have been many investigations into the nature of media over a wide range of temperatures for which noninvasive measurements of various optical coefficients are primary diagnostic tools. For instance, at high temperatures, it is common to use measurements of spectral emission or absorption coefficients for the subsequent calculation of temperatures or concentrations in electric arcs or other plasma systems. At these and lower temperatures, optical interferometry, which measures changes in the index of refraction, is used for subsequent temperature determination in gases and liquids. At still lower temperatures, absorption coefficients in liquids and solids for short-wavelength radiation, such as x- and γ-rays, are measured as a means for determining densities.

In one way or another, many of these experiments involve measurements in an emitting, absorbing, or refracting medium that is variable in both its geometry and local optical characteristics. If the measurements are to be made nonintrusively, then somewhere in the detection and characterization process an extensive property will be measured as a function of an external scanning coordinate. In the examples above, these could include measurements of the volume emission of the total or spectral radiation for temperature and concentration measurement problems, refractive index change for interferometric problems, and volume absorption of radiation for density measurements in two- or three-phase systems. In each case, a distribution of the apparent value of an optical variable will be recorded photographically or electronically for subsequent mathematical manipulation that will yield spatial distributions of the intensive analog of that property. The mechanics of these processes are such that a beam of finite cross section, from either the source itself or some part of the detection system, will be employed. However, the measurements are often used to calculate distributions of the point values of some intensive property. This is the crux of the problem—the resultant point values are not correct for the nominal coordinates with which they are associated.

This paper discusses the effect of a finite-width exit slit on the spatial accuracy of the spectrometric determination of the distribution of the radial emission coefficient for a radiating column with axial symmetry for which the emission coefficient varies from a minimum at the outer radius to a maximum at or near the axis. In particular, models will be used that are often employed in analyzing radiation from plasma streams, electric-arc columns, and flames. In some of these systems, temperature gradients can be thousands of degreess Kelvin per millimeter, so that across the finite aperture of the detection slit or collimating tube there can be not only a variation in the optical depth caused by the geometry of the system, but a variation in the emission coefficient with position.

The Problem

The diagnostics of electric-arc columns for the determination of either temperature or concentration utilize spectrometers, spectrographs, or interference filters to obtain an image or recording of the lateral distribution of radiative flux as seen from a direction normal to the axis of the source. This is shown schematically in Fig. 1. In this kind of experiment, the diagnostic tool can be a spectrometer with an entrance slit oriented in the y direction, so that the narrow width of the entrance slit along with the external focusing optics determines the axial extent h of the slice of radiation from the arc-column entering the spectrometer. At the exit of the spectrometer, there is a spectral image of the radiation entering the entrance slit, which has an intensity distribution in its longitudinal direction (normal to the wavelength) that depends on the lateral (y) distribution of radiant power leaving the source, the external optics of the detector, and the width of the entrance slit. The investigator then either records this intensity distribution directly, using a scanning exit slit and a suitable radiation detector, or photographs the image

Presented as Paper 83-1541 at the AIAA 18th Thermophysics Conference, Montreal, Canada, June 1-3, 1983; received April 2, 1984; revision received Oct. 19, 1984.

*Professor of Mechanical Engineering. Associate Fellow AIAA.
†Graduate Student, Department of Mechanical Engineering. Student Member AIAA.

on a calibrated emulsion from which the distribution of radiative flux can be obtained with a densitometer.

Once the lateral distribution of a flux is obtained for such an experiment, the corresponding distribution of the emission coefficient is obtained by using the Abel integral equation, or its inverted form, directly. It is in the last part of this measurement process, going from the scanning of the spectral image of the light-source volume element to the determination of the emission coefficient, that one may erroneously interpret the final result. This problem arises because of the finite size of the analyzing device used to determine the lateral distribution of radiation flux. In a spectrometer this would be an exit slit, in a densitometer it would the width of the analyzing light beam, and for a fiber-optic probe it would be related to the diameter of the fiber-optic element.

For generality, the aperture will be referred to as the analyzer in what follows. Whatever the case may be, the size of this analyzer along with the other optical elements in the diagnostic instrument will determine the width w shown in Fig. 1. Note that the greater the magnification of the focussing optics (not shown in Fig. 1), the smaller w will be in relation to R so that the error we are addressing will be smaller.

Note that the emission coefficient will vary along the volume element in the x direction with a resultant effect on the optical thickness. However, it will also vary in the y direction, which is the direction across the analyzer in the case of the spectrometer. This is the crux of the problem treated in this paper. Because of this effect, depending on the particular location of the volume element and the nonlinearity of the emission-coefficient distribution, one side of the analyzer will receive a disproportionate amount of radiation as compared with the other. The result is that the calculated distribution of the emission coefficient will be somewhat different than the actual distribution, thereby giving rise to a "probe shift." That is, the measured radiant intensity at the nominal location of the analyzer will correspond in the real case to the radiation that would be determined with an analyzer of infinitesimal width at a somewhat different location. Unfortunately, because of constraints imposed by the intensity of the radiation to be analyzed and the sensitivity of the instruments, this problem clearly becomes more severe the wider the analyzer is made.

There is research reported in the literature where the same kind of results, up to this point, are obtained by photographing the plasma through an interference filter.[1] This gives a monochromatic image of the whole column that is then analyzed with a densitometer having a suitable mask that defines a height such as denoted by h in Fig. 1. Basically, the problem of analysis is no different after that from the one employing a spectrograph. Whatever system is employed, it can be calibrated for either abolute or relative radiation flux measurements. For simplicity in what follows, it will be assumed that relative measurements are employed so that we need not concern ourselves with extraneous optical and geometrical parameters. Also, as h is usually quite small, variations in the z direction will be ignored.

The above problem is quite similar to the "probe shift" that occurs in probing the velocity distribution in a nonlinear velocity profile such as exists in a boundary layer. Here the impact pressure is more representative of the higher velocities on the freestream side of the flow approaching the probe than they are of the lower velocities on the wall side. Whereas the

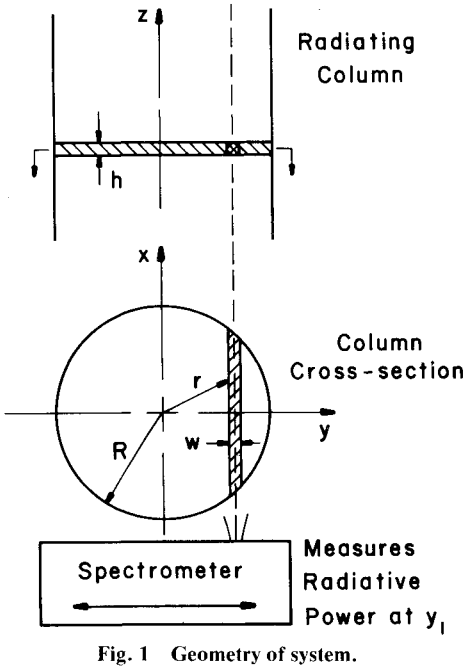

Fig. 1 Geometry of system.

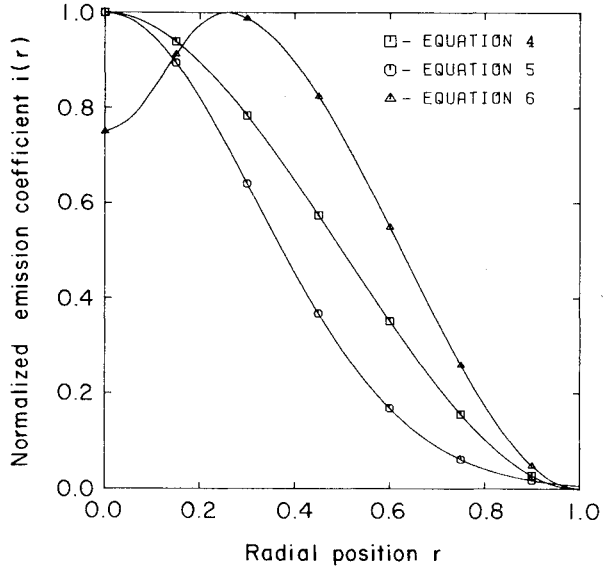

Fig. 2 Test distributions of emission coefficient.

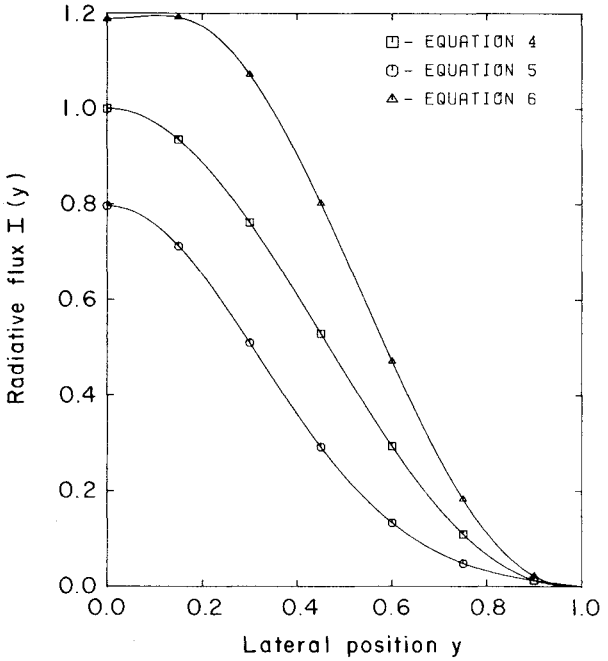

Fig. 3 Test distribution of radiative flux.

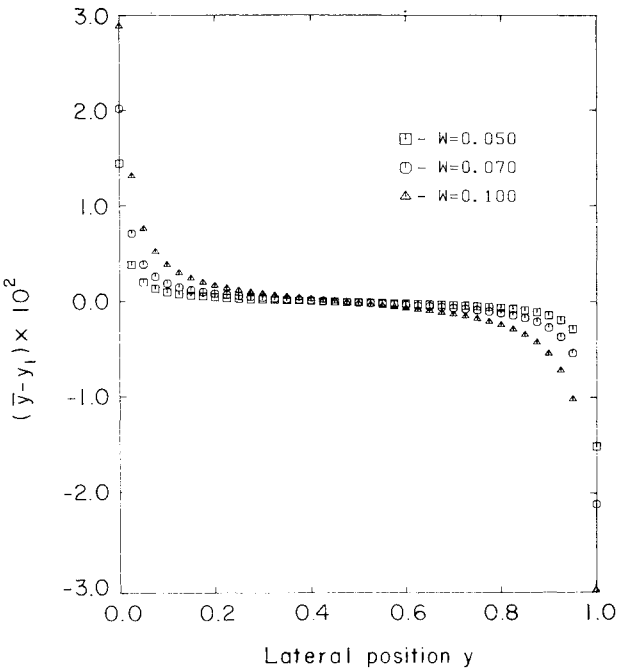

Fig. 4 Probe-shift for *i(r)* of Eq. (4).

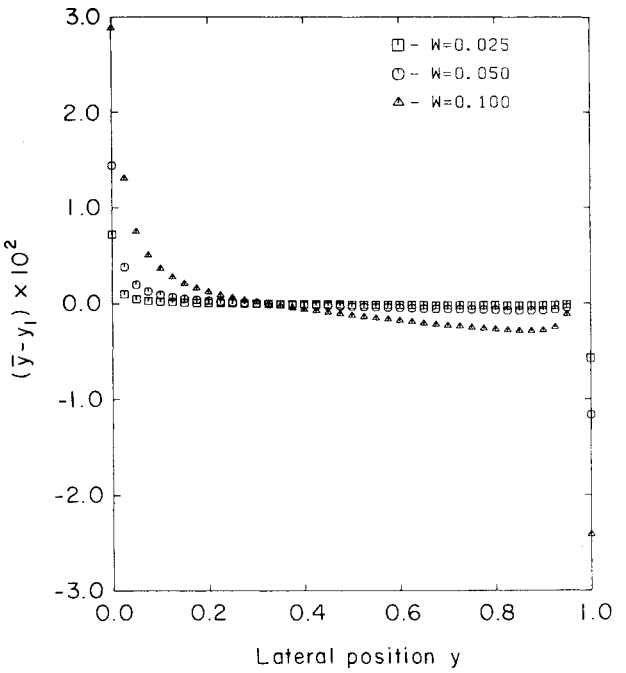

Fig. 5 Probe-shift for *i(r)* of Eq. (5).

"slit function" of a detector such as a spectrometer is a measure of how well a slit of finite width maps spectral radiation in the wavelength direction, the "probe shift" is a measure of how well an analyzer scans the radiation in a lateral direction (direction normal to the axis of the source).

Analysis

Consider the equation describing the radiation flux from a volume source that exhibits cylindrical symmetry. For simplicity, assume that the focusing optics between the source and spectrometer are such that the magnification is unity. Using the geometry of the source shown in Fig. 1, it is easily shown that the flux $I(y)$ emitted in the x direction is given in terms of the emission coefficient $i(r)$ by

$$I(y)=2\int_y^R \frac{i(r)\,r\,dr}{\sqrt{r^2-y^2}} \tag{1}$$

Integrals of this form are called Abel integrals and are special cases of the Volterra equation of the first kind.[2] Usually, in experiments such as described above, $I(y)$ is taken as the measured distribution of radiation flux and the emission coefficient is obtained by taking the integral for each desired y replacing it by a series over finite radial increments, and assuming the emission coefficient to be a constant in each interval. In this way, one obtains a series of equations in which there are as many discreet values of the emission coefficient as there are of the measured $I(y)$.

An alternate approach is to invert Eq. (1) yielding

$$i(r)=-\frac{1}{\pi}\int_r^R \frac{I'(y)}{\sqrt{y^2-r^2}} \tag{2}$$

Here $I'(y)$ is the first derivative of the radiation flux taken with respect to the lateral coordinate y. In this form, the emission coefficient at each desired r can be obtained either by again using finite increments and a differencing scheme to approximate the derivative for each increament or by approximating the measured lateral flux with a suitable analytic expression that can then be differentiated so that the integral can be solved directly.

If now one considers the effect of an analyzer of finite width, a radiative flux is measured that is a function of that width. Therefore, if $I(y_I)$ is the true value of measured flux at the nominal position of the measurement (y_I), then what is measured for the volume element is a radiative power $A(y_I,w,h)$ that is given by

$$A(y_I,w,h)=h\int_{y_I-w/2}^{y_I+w/2} I(y)\,dy \tag{3}$$

$A(y_I,w,h)$ is usually interpreted as being a measure of $I(y_I)$, whereas in reality, because of the nonlinear distribution of $i(r)$, it corresponds to the radiation flux $I(\bar{y})$ that one would measure at some nearby coordinate $\bar{y}$, if the measurement had been made with an infinitesimal analyzer width. In other words, the radiative flux determined at the nominal position y_I should be attributed to $\bar{y}$ and the difference $(\bar{y}-y_I)$ can be interpreted as the "probe shift."

It is a tedious task to establish the probe shift for a particular set of experimental data. It is necessary to fit either a suitable analytic expression to the data or to work the problem numerically in its entirety. For purposes of the present discussion, distributions of the normalized emission coefficient that well approximate arc-columns operating under several different conditions will be used to illustrate the problem. These are[3]

$$i(r)=1-3r^2+2r^3, \qquad (0\le r\le 1) \tag{4}$$

$$i(r)=\exp(-r^2/\sigma^2), \qquad (\sigma=0.45,\ 0\le r\le) \tag{5}$$

and

$$i(r)=3/4+12r^2-32r^3 \qquad (0\le r\le 0.25)$$

$$=16/27\,(1+6r-15r^2 8r^3) \qquad (0.25\le r\le) \tag{6}$$

The simple polynomial of Eq. (4) and the Gaussian distribution of Eq. (5) are approximations of low-temperature arcs (<15,000 K for a 1 atm Ar arc) and the polynomials of Eq. (6) are representative of a column in which the radiating species has been severely depleted by ionization in the higher-

temperature zones. These distributions are shown for comparison in Fig. 2 and are used with Eq. (1) to develop the corresponding distributions of radiative flux $I(y)$ that are shown in Fig. 3. In turn, these will be used with Eq. (3) to generate a set of $A(y_l,w,h)$ that would correspond to the radiative power measured with an analyzer of width w centered at lateral location y_l. Up to this point, the problem is being worked backwards when compared with an actual experiment problem where $A(y_l,w,h)$ is measured for each y_l, either continuously or discretely.

The quantity $A(y_l,w,h)$ is normally interpreted as a measure of the radiative flux at the position y_l; for a very narrow analyzer, this assumption is well founded. However, as mentioned above, because of the nonlinear distribution of radiation across the analyzer caused by the geometry of the system as well as by the emission coefficient distribution, the flux determined in this way is actually representative of some other location, the effective lateral coordinate $\bar{y}$. This flux can be calculated as

$$B(\bar{y}) = A(y_l,w,h)/wh \tag{7}$$

and in turn can be intepreted as

$$B(\bar{y}) = 2\int_{\bar{y}}^{R} \frac{i(r)\,r\,dr}{\sqrt{r^2-y^2}} \tag{8}$$

Equation (8) was solved for the $\bar{y}$ distributions of the three cases above by using the Newton-Raphson method.

The problem of incorrectly attributing the measured emission coefficient to the nominal location of the slit can be looked at somewhat differently by determining what the measured value would really be at the nominal location had the analyzer been of infinitesimal width. To do this the emission coefficient is obtained from Eq. (2) written in the form

$$i(r) = -\frac{1}{\pi}\int_{r}^{R} \frac{B'(y)\,dy}{\sqrt{y^2-r^2}} \tag{9}$$

Here the derivative $B'(y)$ is obtained by fitting cubic interpolating splines to the $B(\bar{y})$ distribution and differentiating the result. Equation (9), then, yields the distribution of emission coefficient that would be measured for the data set, but is incorrect because of the effect of analyzer width. For the cases at hand, the exact values from Eqs. (4-6) are subtracted from these hypothetical measured values for each y. The result is a measure of the error in emission coefficient as caused by the finite width of the analyzer.

Results and Discussion

The probe shift given by $(\bar{y}-y_l)$ is shown in Figs. 4-6 for a unit source radius and several representative values of the

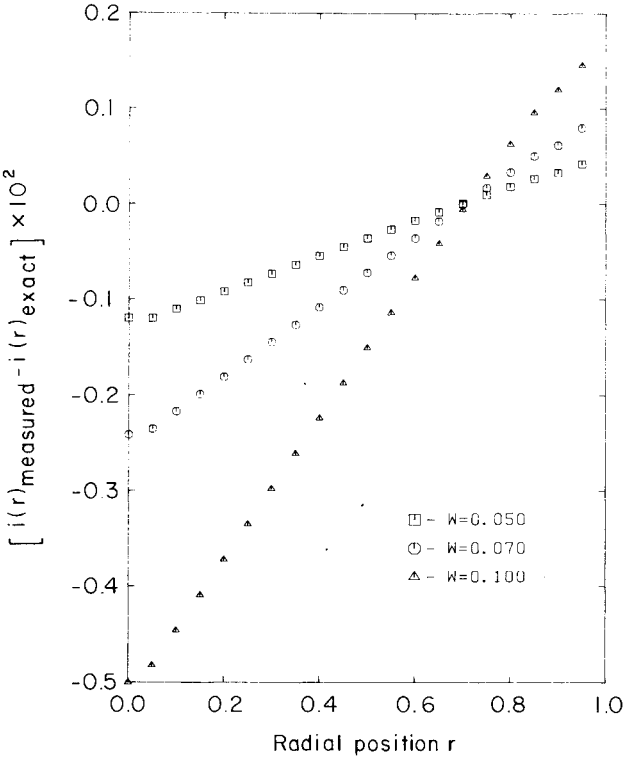

Fig. 7 Error in measured emission coefficient of Eq. (4).

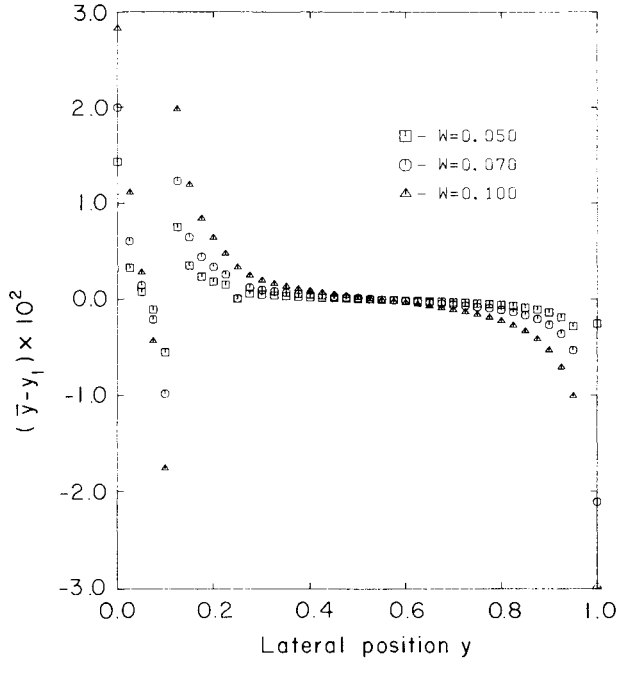

Fig. 6 Probe-shift for *i(r)* of Eq. (6).

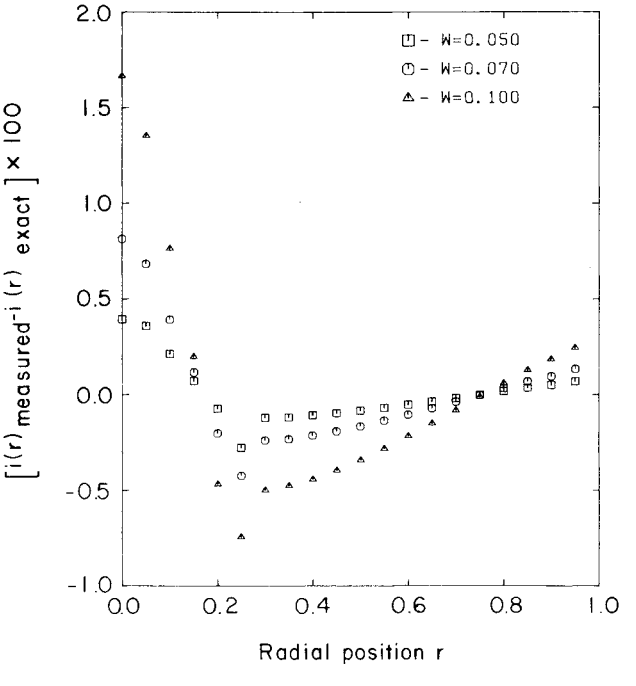

Fig. 8 Error in measured emission coefficient of Eq. (5).

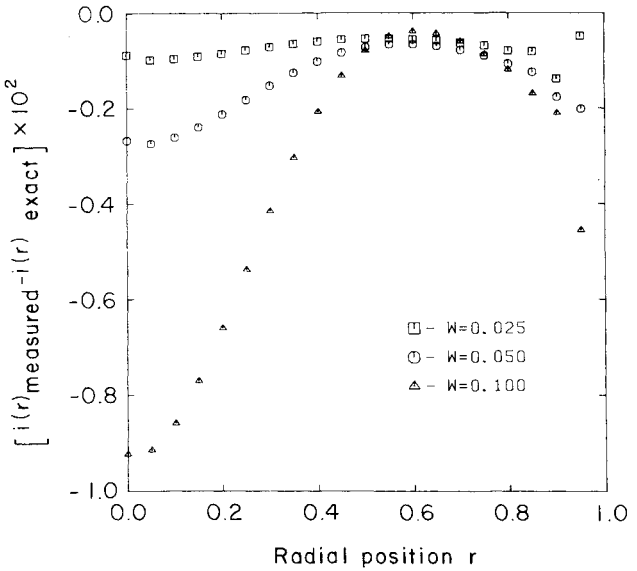

Fig. 9 Error in measured emission coefficient of Eq. (6).

analyzer width w. The latter is given as a fraction of the former. Note that in all cases the shift is worse at large and small radii than at intermediate values and also that it passes through one or more changes. Comparison of these figures with the respective curves for the emission coefficient shown in Fig. 2 indicates that the magnitude and sign of the shift is a strong function of the magnitude and sign of the second derivative of the emission coefficient distribution. As would be expected, the shift increases as w increases.

Figures 7-9 show the difference between the emission coefficient that would be obtained by direct measurement using a finite analyzer width and the true value for the corresponding position y_1. The former is calculated from Eq. (9). As with the probe shift, this difference increases with w and is worst where the gradients are changing fastest.

The application of the ideas presented above to real measurements could easily be done on a case-by-case basis. If the measured lateral distributions of radiative power were amenable to approximation by polynomials or other analytic functions, as in the examples above, then one could use the same procedures as presented. With typical experimental data, however, it is often difficult to fit appropriate curves to the data and one must instead do a numerical solution of the problem wherein the various integrals are replaced by series approximations. One could also apply the correction procedure to asymmetric sources as well. In this case, the entire problem is more difficult as the lateral distribution of radiative flux must be determined from several directions.[4]

Conclusions

We have shown that there is a positioning error involved in the determination of emission coefficients by analyzing radiative power distributions obtained by observing an axisymmetric source from the side with a spatial scanning analyzer of finite width and then using the Abel integral equation for subsequent determination of the emission coefficient. The magnitude and sign of both the positioning error for a particular emission coefficient or the error in the emission coefficient for a particular location depend respectively on the geometry of the source and measuring system as well as the shape of the emission coefficient distribution and analyzer width.

Acknowledgment

This research has been partially sponsored by the National Science Foundation under Grant CME-8005676.

References

[1] Benenson, D. M., Baker, A. J., and Cenkner, A. A., Jr., "Diagnostics on Steady-State Cross-Flow Arcs," *IEEE Transactions on Power Apparatus* and Systems, Vol. 88, May 1969, pp. 513-521.

[2] Hildebrand, F. B., *Methods of Applied Mathematics*, Prentice Hall, Englewood Cliffs, N.J., 1952.

[3] Cremers, C. J. and Birkebak, R. C., "Application of the Abel Integral Equation to Spectrographic Data," *Applied Optics*, Vol. 5, No. 6, 1966, pp. 1057-1064.

[4] Freeman, M. P. and Katz, S., "Determination of a Radiance Coefficient Profile from the Observed Asymmetric Radiance Distribution of an Optically Thin Radiating Medium," *Journal of the Optical Society of America*, Vol. 53, No. 10, 1963, pp. 1172-1179.